Determine if the following series converge, converge absolutely or diverge.

(a) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{10}\right)^n$ (b) $\sum_{n=1}^{\infty} \sqrt{\frac{n+4}{n^4+5}}$

(c) $\sum_{n=1}^{\infty} \frac{1}{1^2+2^2+\cdots+n^2}$ (d) $\sum_{n=2}^{\infty} \frac{(n(n+1)}{n+1}$ (e) $\sum_{n=1}^{\infty} \frac{n-2}{n^3-n^2+3}$.

(a) $\lim_{n\to\infty} (-1)^{n+1} \left(\frac{n}{10}\right)^n$ diverges to $\pm\infty$ and thus, by the n^{th} term test, the series diverges.

(b) $\sum_{n=1}^{\infty} \sqrt{\frac{n+4}{n^4+5}}$ looks most similar to $\sqrt{\frac{n}{n^4}} = \frac{1}{n^{3/2}}$.

Using the Limit Comparison Test

$$\lim_{n\to\infty} \frac{\sqrt{\frac{n+4}{n^4+5}}}{1/n^{3/2}} = \lim_{n\to\infty} \frac{\sqrt{n+4}}{\sqrt{n^4+5}} \cdot \sqrt{n^3} = \lim_{n\to\infty} \frac{\sqrt{n^4+4n^3}}{\sqrt{n^4+5}} = 1.$$

As $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges, so must $\sum_{n=1}^{\infty} \sqrt{\frac{n+4}{n^4+5}}$.

Since the sequence is positive, this is absolute convergence.

(c) Clearly $\frac{1}{1^2+2^2+\cdots+n^2} \le \frac{1}{n^2}$. By the Direct Comparison Test

$$\sum_{n=1}^{\infty} \frac{1}{1^2+2^2+\cdots+n^2} \text{ converges absolutely.}$$

(d) $\sum_{n=2}^{\infty} \frac{\ln(n+1)}{n+1} = \sum_{n=3}^{\infty} \frac{\ln(n)}{n}$. Since $\frac{\ln(n)}{n} \geq \frac{1}{n}$ $\forall n \geq 3$,

the Direct Comparison Test says $\sum_{n=2}^{\infty} \frac{\ln(n+1)}{n+1}$ diverges.

(e) $\sum_{n=1}^{\infty} \frac{n-2}{n^3-n^2+3}$ looks most like $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Using the Limit Comparison Test,

$$\lim_{n\to\infty} \frac{\frac{n-2}{n^3-n^2+3}}{1/n^2} = \lim_{n\to\infty} \frac{n-2}{n^3-n^2+3} \cdot \frac{n^2}{1} = \lim_{n\to\infty} \frac{n^3-2n^2}{n^3-n^2+3} = 1$$

and thus the series converges absolutely.